Practical Applications of STP-Based Logical Networks in Automotive Powertrain Control Design

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Prologue

Approach to Control Practice with Logical Network Framework

Physics: Continues Domain revolution in Discrete-time

Modeling: Quantitative representation of the State

Description with Logical variables

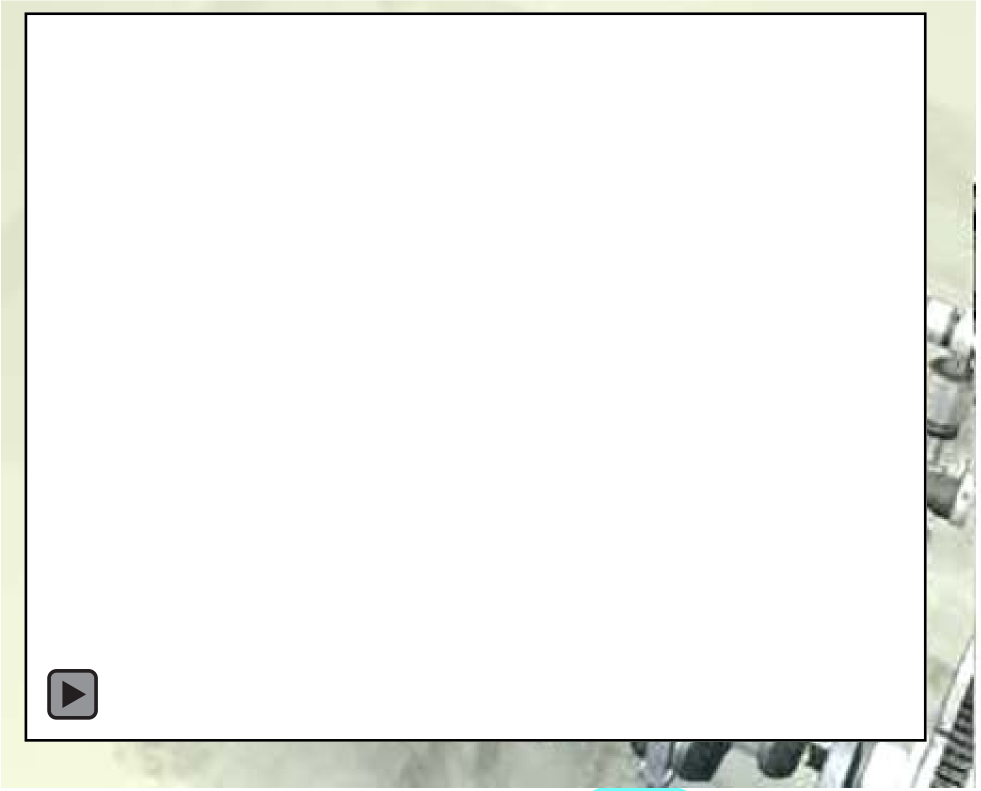
Why:

Essentially Stochastic system

Controlling precise value doesn't make sense

Low sensitivity of actuator

Combustion Engine

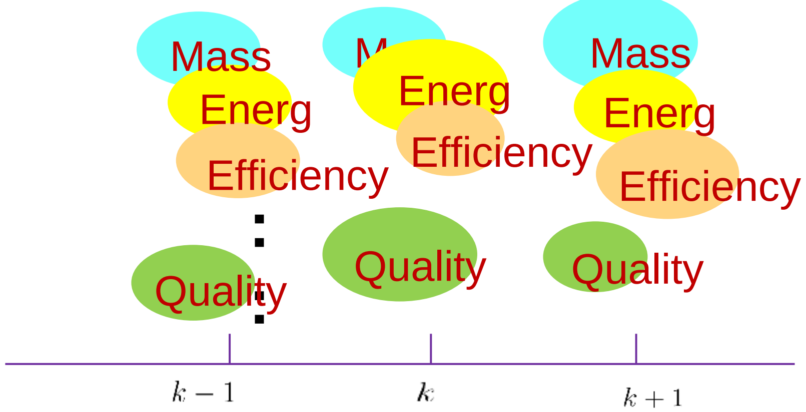


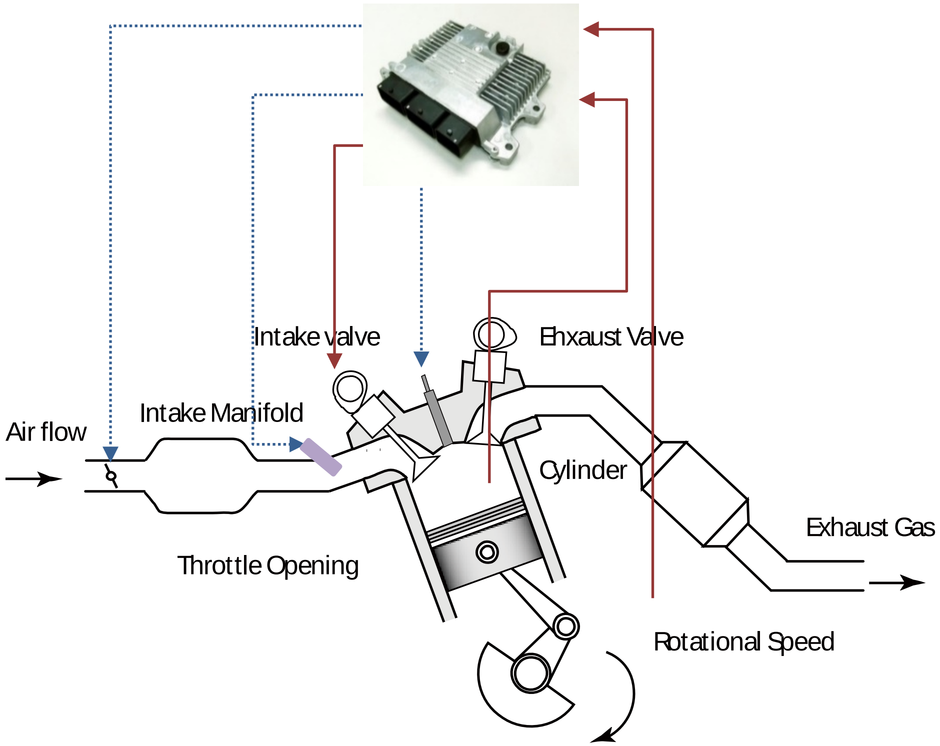
- Cycle-to-cycle revaluation

Discrete-time

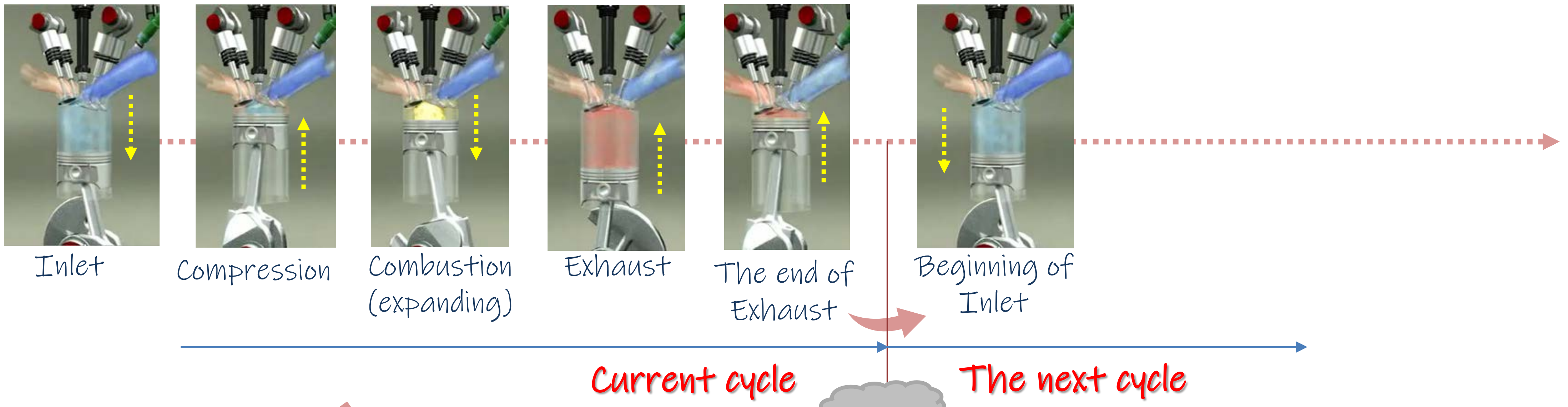
- Stochasticity in Combustion

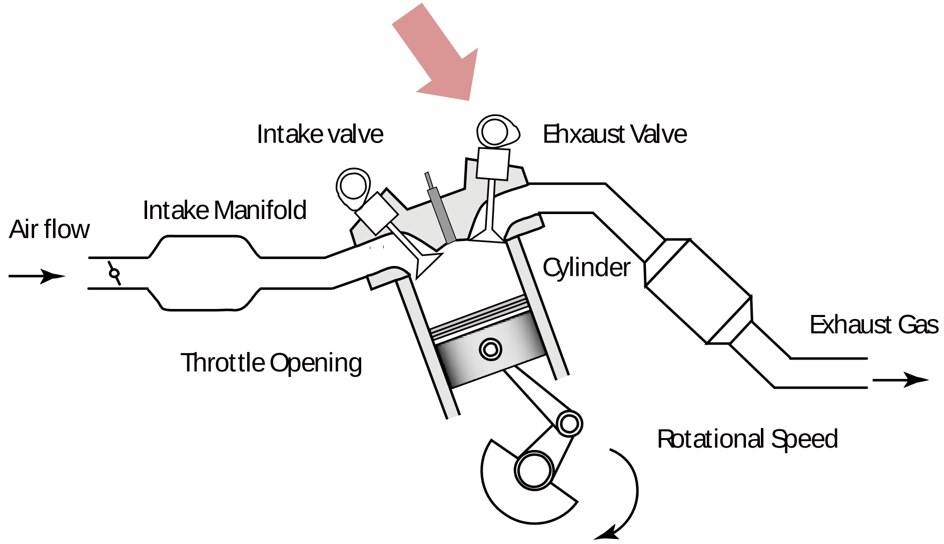
Continuous variable





The RGF: internal exhaust gas recirculation



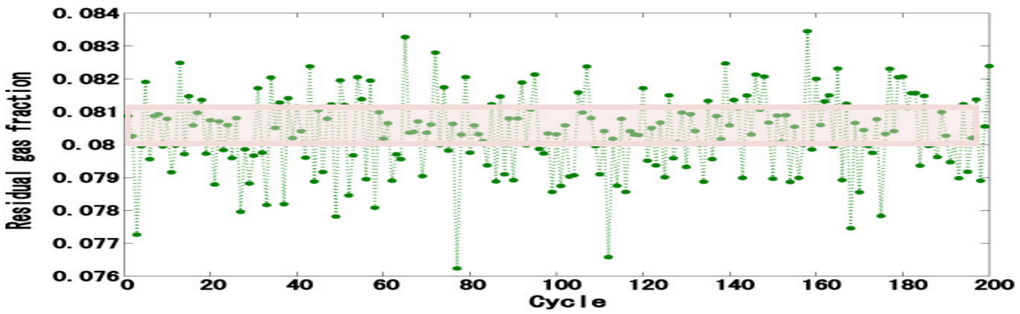


The residual gas (burnt and unburn) goes to the next cycle

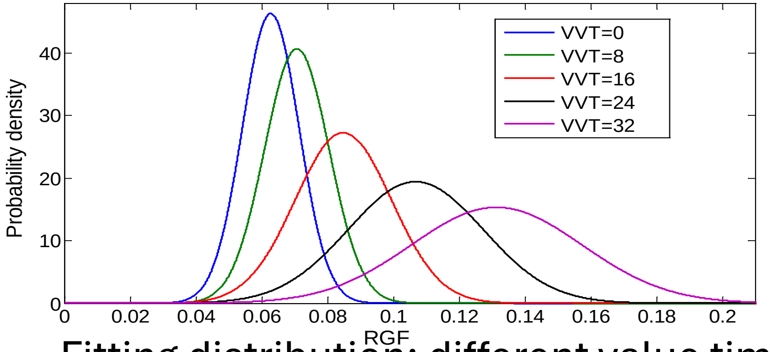
The residual gas fraction (RGF)

Mass of RG/ Total gas mass
=10%±Delta

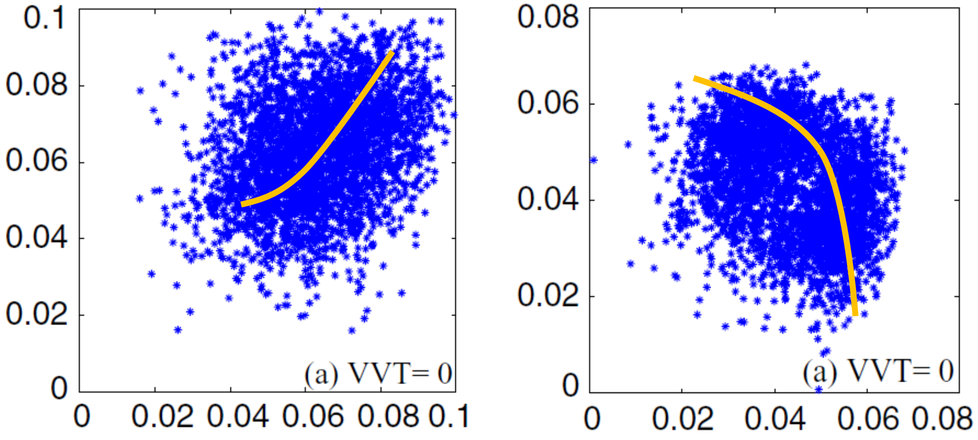
Physics: Stochasticity



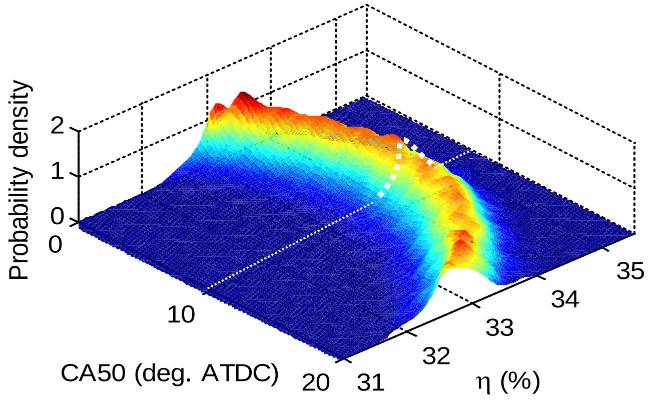
Cycle-to-cycle variation under fixed actuator value



Fitting distribution: different value timing



Kth-cycle vs k+1th cycle



Stochasticity in combustion event

Modeling: $\Pr\{x_{k+1} = s_j\} = T(x_k, u_k, \omega_k) \Leftarrow x_{k+1} = f(x_k, u_k, w_k)$

- Derivation of Optimal Control Policy
- Example i:

  RGF control of Combustion Engines
- Example ii:

  Energy management strategy design for HEVs

Developing an Design Method

Stochastic Logical Networks: System Description

State space

$$S = \left\{ \begin{bmatrix} 1\\0\\0\\0\\0\\0\\0 \end{bmatrix}, \begin{bmatrix} 0\\1\\0\\0\\0\\0\\0 \end{bmatrix}, \cdots, \begin{bmatrix} 0\\0\\0\\0\\0\\0\\1 \end{bmatrix} \right\}$$

s

Control space

$$U = \left\{ \begin{bmatrix} 1\\0\\0\\0\\0 \end{bmatrix}, \begin{bmatrix} 0\\1\\0\\0\\0 \end{bmatrix}, \cdots, \begin{bmatrix} 0\\0\\0\\0\\1 \end{bmatrix} \right\}$$

- δ_s^i : i-column of the identity matrix I_s.

Dynamics

$$x_{k+1} = f(x_k, u_k, w_k)$$

logical operator

External random disturbances

For example:

$$S = U = \left\{ \begin{bmatrix} 1\\0 \end{bmatrix}, \begin{bmatrix} 0\\1 \end{bmatrix} \right\}$$

$$f(x, u, w) = (x \wedge u \neg w)$$

$$f(\delta_2^1, \delta_2^2, \delta_2^2) = \delta_2^2$$

$$f(\delta_2^2, \delta_2^1, \delta_2^1) = \delta_2^1$$

[1] **D. Cheng**, H. Qi, and Z. Li, *Analysis and control of Boolean networks: a semi-tensor product approach*. Springer, 2011.

Stochastic Logical Networks: Optimal Control

- Admissible policy (reachable logical set)

$$\Pi = \{\mu_0, \mu_1, \cdots, \mu_{N-1}, \cdots\}, \qquad \mu_k : S \to U, k = 0, 1, \cdots, N-1.$$

- Cost Index

$$J_\pi(x_0) = \underset{\substack{w_k \\ k=0,1,\cdots}}{E} \sum_{k=0}^{N} \alpha^k \underbrace{g(x_k, u_k)}_{}, \qquad \boxed{\text{per stage cost}}$$

- Finite Horizon problem

Find the optimal policy $\pi^* \in \Pi$ s.t.

$$J_{\pi^*}(x_k) = J^*(x_k) \triangleq \inf_{\pi \in \Pi} \underset{\substack{w_k \\ k=0,1,\cdots}}{E} \sum_{k=0}^{N-1} \alpha^k g(x_k, u_k),$$

- Infinite Horizon problem

$$J_{\pi^*}(x_k) = J^*(x_k) \triangleq \inf_{\pi \in \Pi} lim_{N\to\infty} \underset{\substack{w_k \\ k=0,1,\cdots}}{E} \sum_{k=0}^{N-1} \alpha^k g(x_k, u_k),$$

Algebraic Representation

For a control law $\mu \in \mathcal{U}$, the transition probability matrix P_μ is defined by

$$P_\mu = \begin{pmatrix} p_{11}(\mu(\delta_s^1)) & \cdots & p_{1s}(\mu(\delta_s^1)) \\ \vdots & \vdots & \vdots \\ p_{s1}(\mu(\delta_s^s)) & \cdots & p_{ss}(\mu(\delta_s^s)) \end{pmatrix},$$

and the cost vector g_μ is defined by $g_\mu = (g(\delta_s^1, \mu(\delta_s^1)), \cdots, g(\delta_s^s, \mu(\delta_s^s)))^T$.

Proposition A

For any control law $\mu \in \mathcal{U}$, the transition probability matrix P_μ associated with control law μ can be calculated by

$$P_\mu = M_\mu \mathbb{P},$$

where the matrix $M_\mu \in M_{s\times(sr)}$ is defined by

$$M_\mu = \begin{pmatrix} (\delta_s^1)^T \ltimes (\mu(\delta_s^1))^T \\ (\delta_s^2)^T \ltimes (\mu(\delta_s^2))^T \\ \vdots \\ (\delta_s^s)^T \ltimes (\mu(\delta_s^s))^T \end{pmatrix}.$$

Transition probability matrix of "Closed Loop System under Control ¥mu" can be generated by multiplying a matrix

Optimal Control—Policy Iteration

Definition

We define a hyperplane $\mathbb{D}^{s+1}$ of R^{s+1} as $\mathbb{D}^{s+1} = \{(x_0, x_1, \cdots, x_s) \in R^{s+1} : x_0 = 1\}$. For any control law $\mu \in \mathcal{U}$, we define operator $Q_\mu : \mathbb{D}^{s+1} \to \mathbb{D}^{s+1}$ as

$$Q_\mu x = \begin{pmatrix} 1 & 0 \\ g_\mu & \alpha P_\mu \end{pmatrix} x, \quad \forall x \in \mathbb{D}^{s+1}, \tag{1}$$

and define operator $Q : \mathbb{D}^{s+1} \to \mathbb{D}^{s+1}$ as

$$[Qx]_i = \inf_\mu [Q_\mu x]_i, \quad \forall i = 1, \cdots, s+1, \quad x \in \mathbb{D}^{s+1}. \tag{2}$$

Equivalently, this is nothing but minimization of "one step" in Bellman DP

- $\Delta_s := \{\delta_s^i | i = 1, 2, \cdots, s\}$.
- $S \sim \Delta_s \iff x^i \sim \delta_s^i, \quad i = 1, 2, \cdots, s,$
- $U \sim \Delta_r \iff u^j \sim \delta_r^j, \quad i = 1, 2, \cdots, r.$

Optimality Condition

Proposition B

For any control law $\mu : \Delta_s \to \Delta_r$, for any vector $J \in R^s$, the operator $Q_\mu : \mathbb{D}^{s+1} \to \mathbb{D}^{s+1}$ and the corresponding cost vector $J_\mu \in R^s$ of μ satisfies

$$\lim_{N\to\infty} Q_\mu^N \overline{J} = \overline{J}_\mu. \tag{3}$$

Proposition A

A stationary policy μ is optimal if and only if

$$Q\overline{J}^* = Q_\mu \overline{J}^*, \tag{4}$$

where J^* is the vector form of optimal cost,

$$J^* = \left(J^*(\delta_s^1), J^*(\delta_s^2), \cdots, J^*(\delta_s^s)\right)^T.$$

[2] **Yuhu Wu**, and **Tielong Shen**, A finite convergence criterion for the discounted optimal control of stochastic logical networks, ***IEEE Transactions on Automatic Control***, *2017*.

Optimal Control—Policy Improvement

Algorithm

The optimal control problem can be solved as follows.
Step 0. Initialization: Guess an initial policy $\mu^0 \in \mathcal{U}$.
Step 1. Policy Evaluation: For given stationary policy μ^k, compute the corresponding J_{μ^k} from

$$Q_{\mu^k}\overline{J}_{\mu^k} = \overline{J}_{\mu^k}. \tag{1}$$

Step 2. Policy Improvement: Obtain a new stationary policy μ_{k+1} by $\mu^{k+1}(x) = \Phi_{k+1}x, \ \ \forall\, x \in \Delta_s$, where the structure matrix Φ_{k+1} of μ^{k+1} is calculated by

$$\begin{cases} \Phi_{k+1} = L_r[q_1^{k+1}, \cdots, q_s^{k+1}], \text{ with } i = 1, \cdots, s, \\ q_i^{k+1} = {}_{j=1,\cdots,r} \left\{ G_{ij} + (\delta_s^i)^T \ltimes (\delta_r^j)^T \mathbb{P} J_{\mu^k} \right\}. \end{cases}$$

If $\overline{J}_{\mu^k} = Q\overline{J}_{\mu^k}$, then the process is terminated; otherwise return to Step 2 and repeat the process.

Short Summary on Algorithm Developing

☆ Transition Probability matrix of CLS

➡ multiply a policy-associated matrix M to basis vector

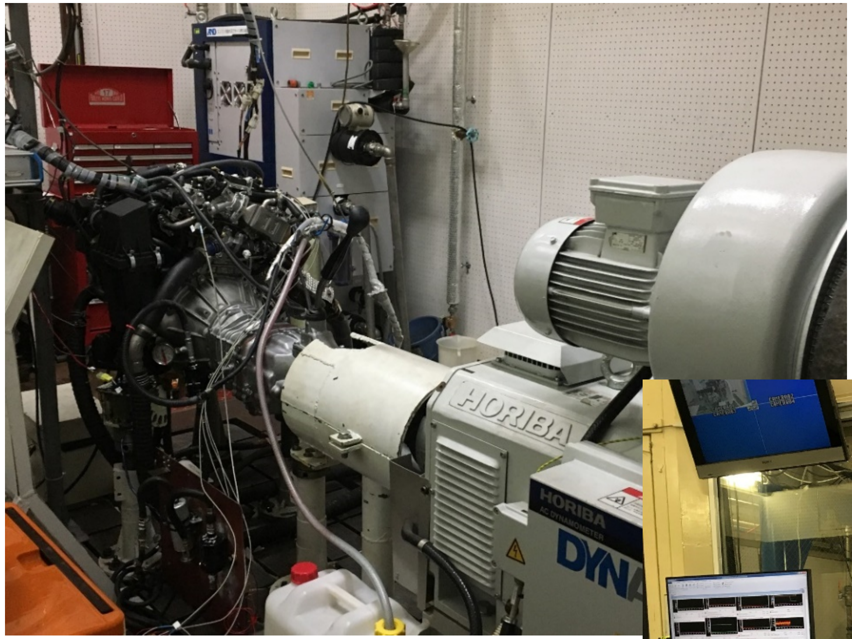
☆ Benefit from the definition of Q_¥mu matrix

➡ found that the optimal cost J^* is a "fixed-point" of Q!

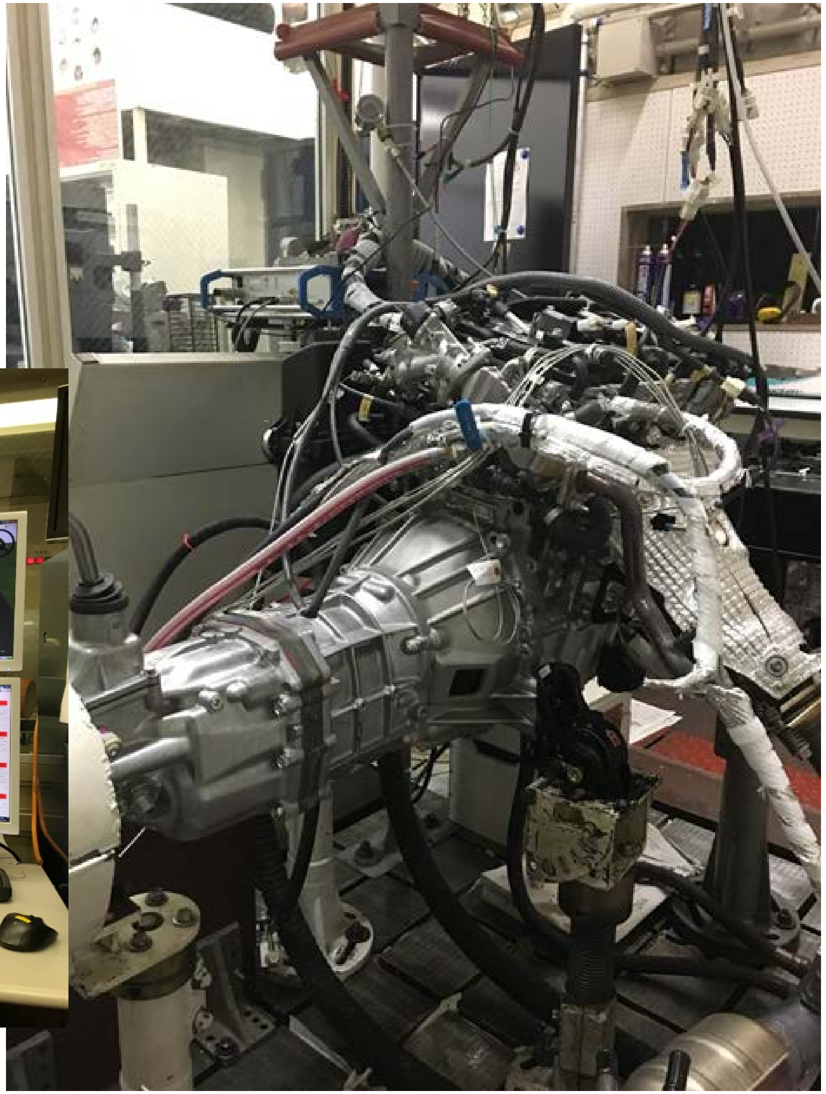
➡ provides a condition "policy evaluation" which enables us to construct a policy iteration algorithm.

Example of the Combustion Engine

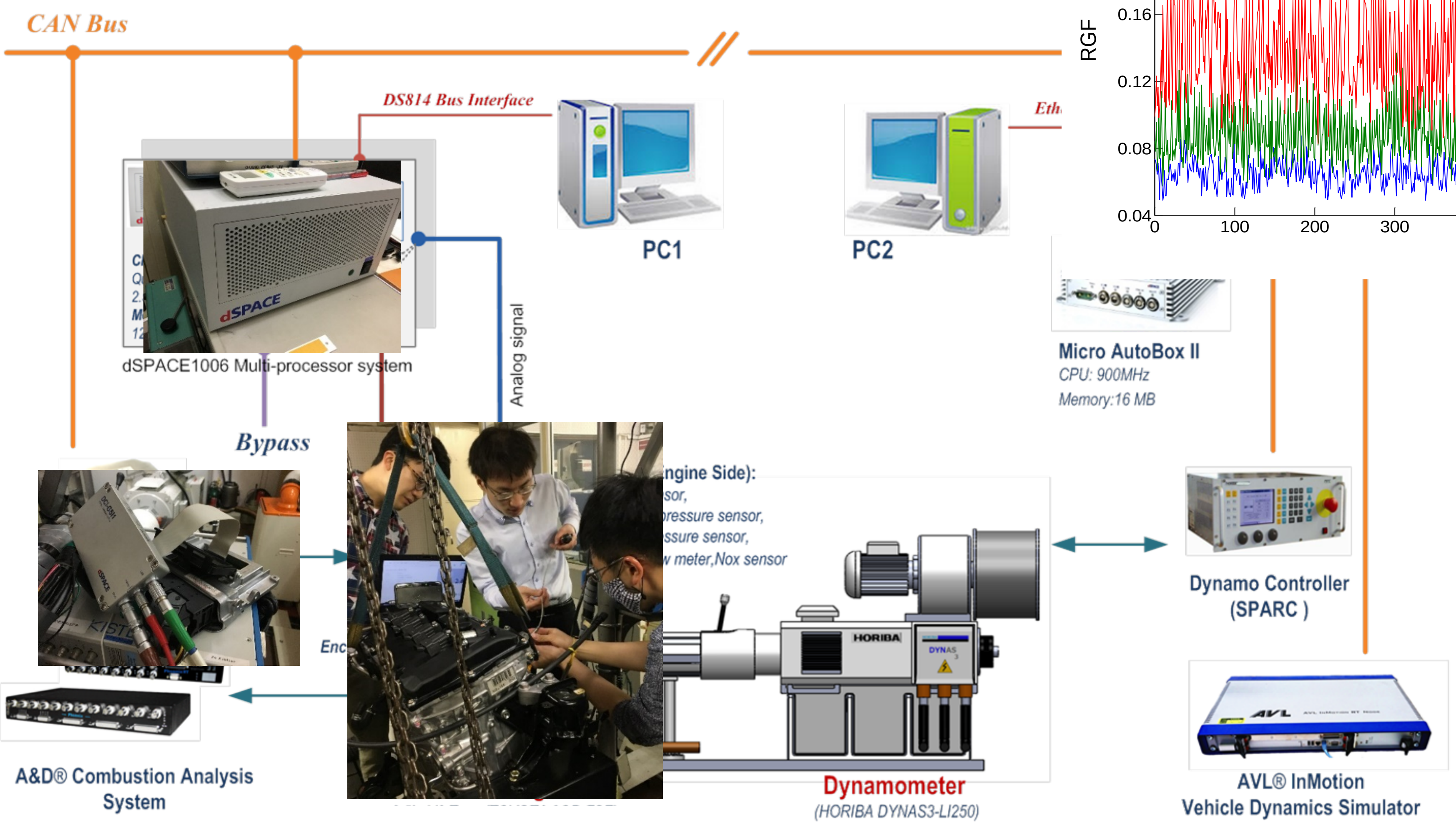
Testbench: getting the physics

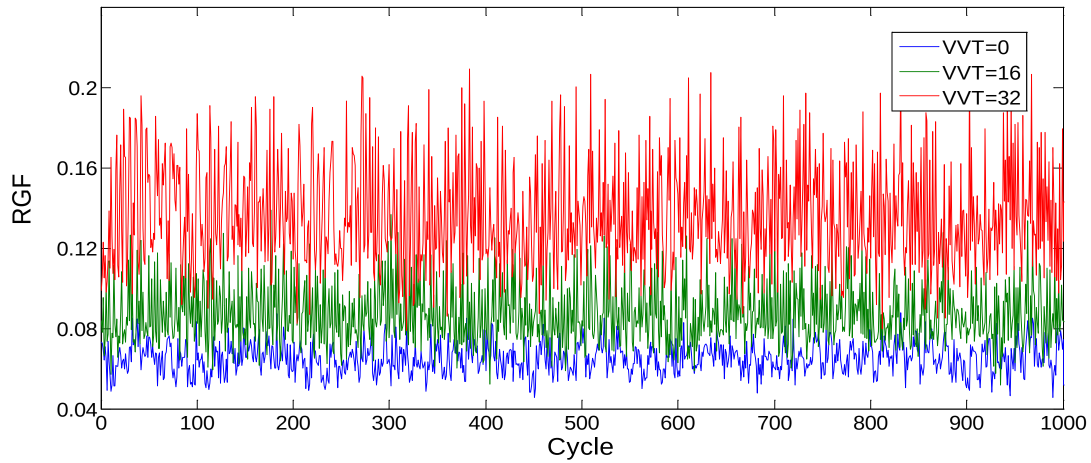


Engine type	2ZR-FXE
Displacement	1797ml
Compression ratio	13.0
Maximal power	72[kW]
Maximal torque	142[Nm]



Testbench: Measuring and controlling

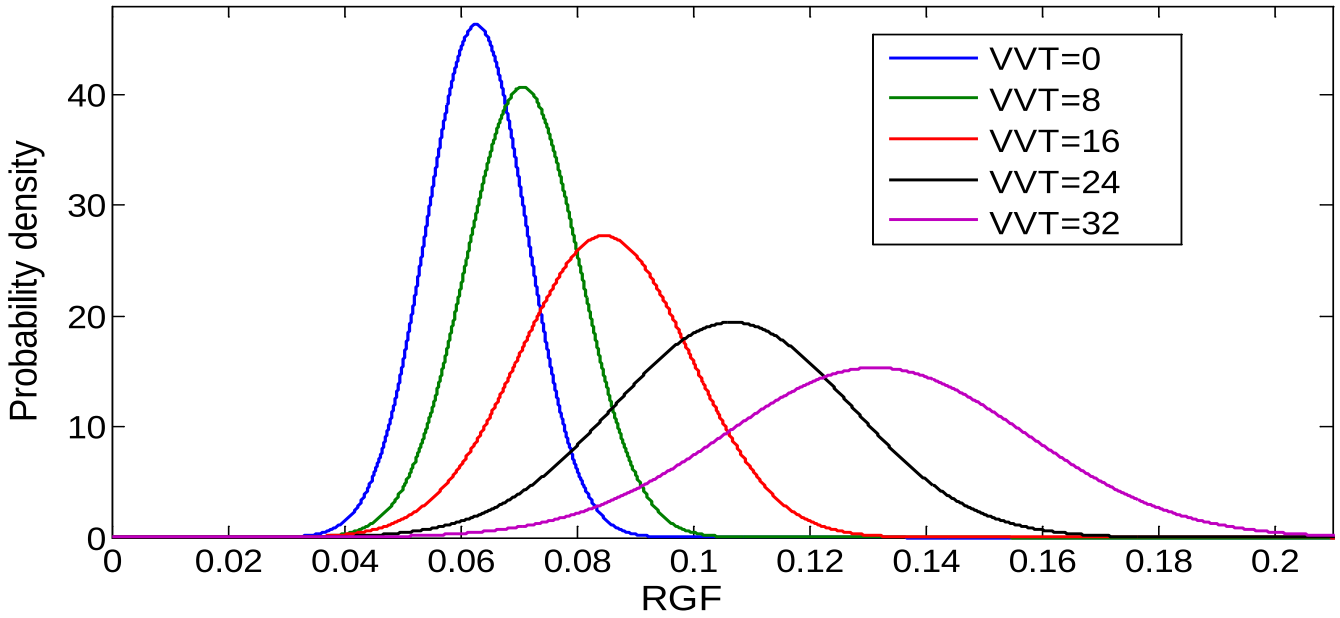




Modeling as logical system

The statistic properties of RGF displayed in Fig. 3 implies VVT evidently affects the distribution of RGF.

VVT can be regard as the control input with three discrete degrees 0, 8,16,24, 32.

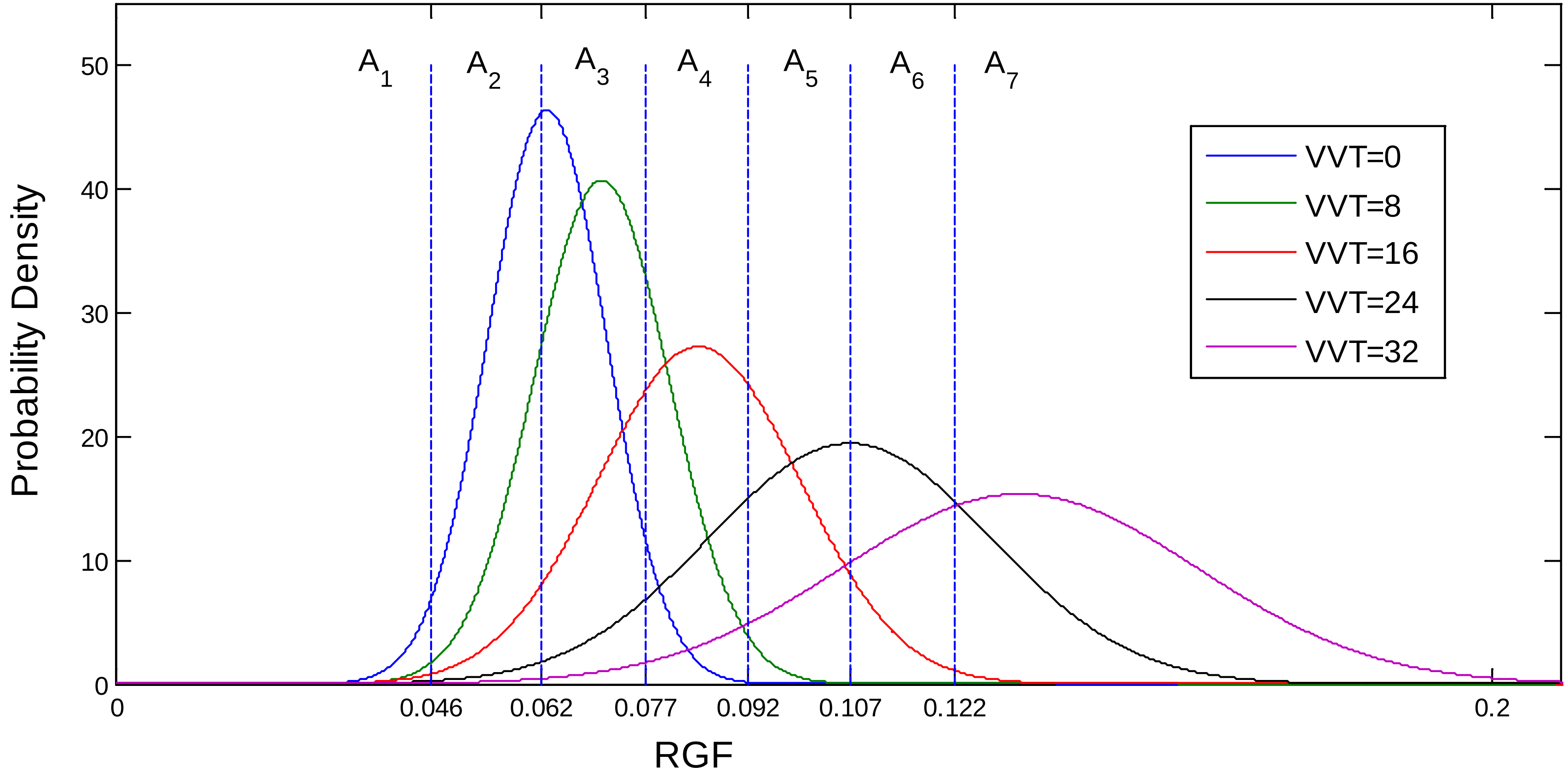


The probability density of RGF under various constant VVT.

	Mean Value	Standard Variation
VVT=0	0.0628	0.0086
VVT=8	0.0707	0.0098
VVT=16	0.0849	0.0146
VVT=24	0.1069	0.0205
VVT=32	0.1316	0.0260

Modeling: Quantization & Logical System Representation

The range of RFG is divided into seven intervals $A^1, A^2, \cdots, A^7$, where

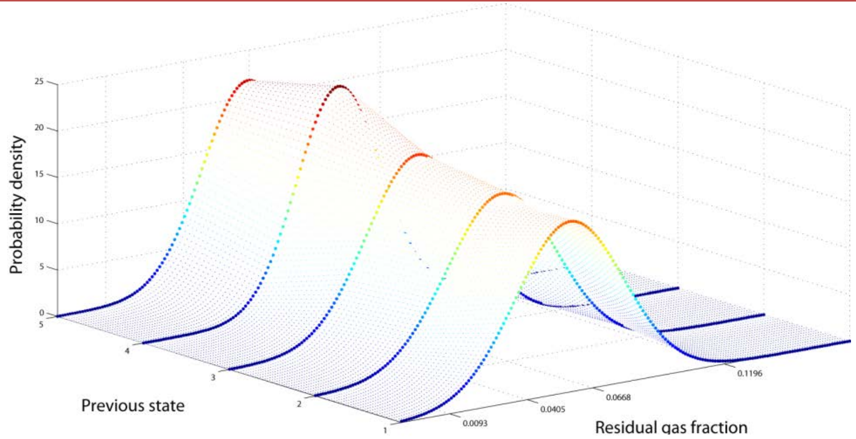


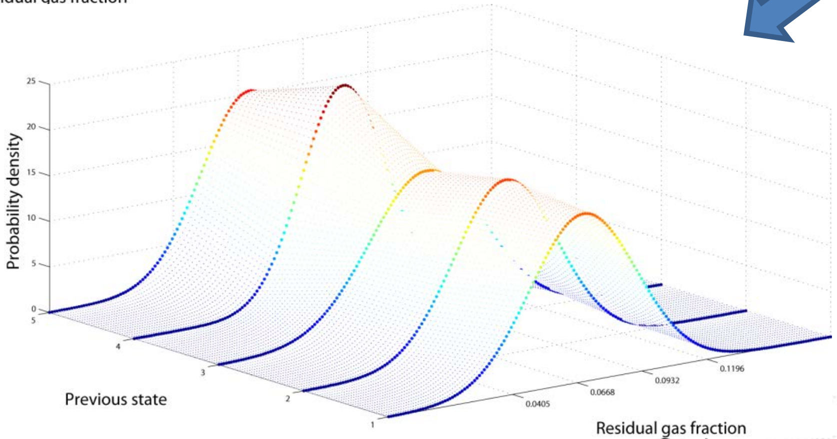
The values of RGF are quantified in following identification way:

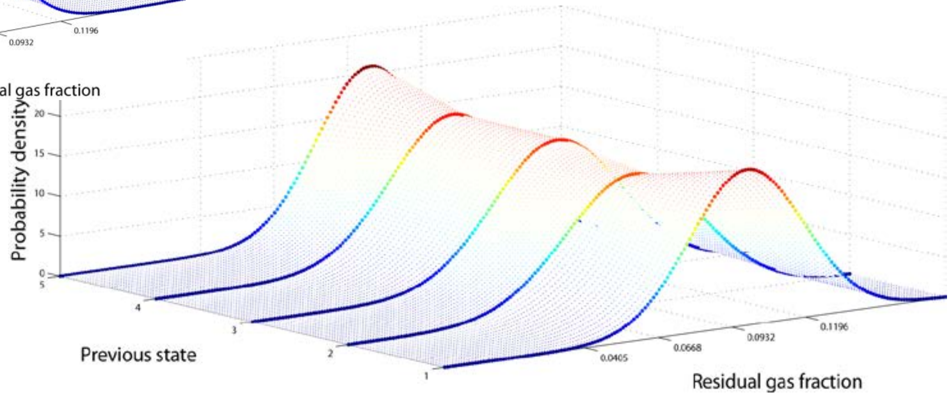
$$y_t \in A^i \leftrightarrow x_t = \delta_7^i, \ \ i = 1, 2, \ldots, 7.$$

Fitting Transition Probability under Control

$$VVT \overbrace{0, 16, 32}$$







Conditional probability density under different previous states and controls

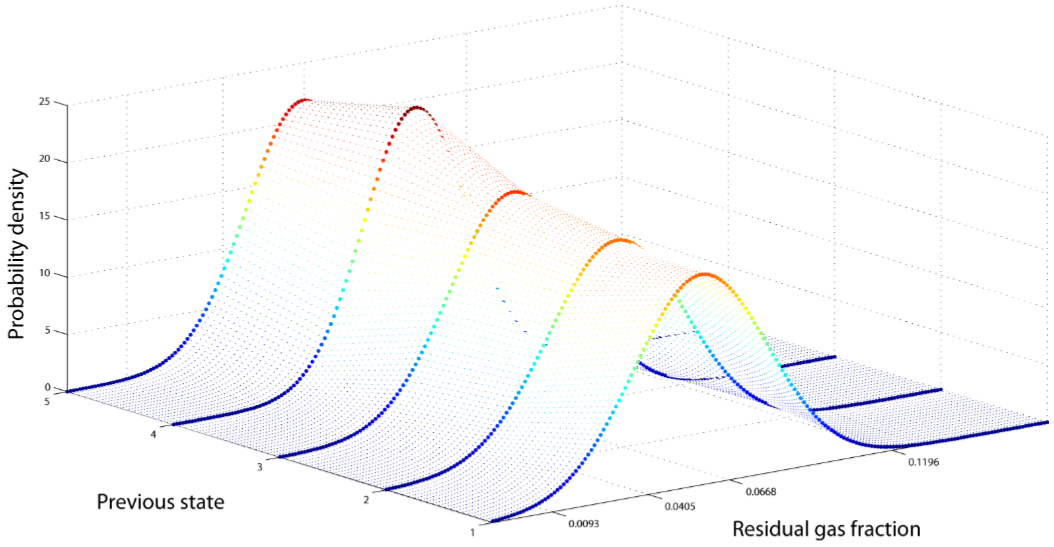
Fitting Transition Probability under Control

The conditional probability density for Residual Gas Fraction

$$Y_{i,t} = \{y_{t+1} | x_t \in A^i, u_t = \delta_5^k\},$$
$$i = 1, 2, \cdots, 7, k = 1, 2, , \cdots, 5.$$

Maximum Likelihood Estimation

$$Y_{i,k} \sim N(\mu_{i,k}, \sigma_{i,k}^2).$$



$$P(y_{t+1} | y_t \in A^i, \boxed{u_t = \delta_5^4})$$

Then the transition probabilities can be calculated

$$P_{ij}(\delta_r^k) = \int_{X_j} \frac{1}{\sqrt{2\pi}\sigma_{i,k}} \exp\left(-\frac{(y-\mu_{i,k})^2}{2\sigma_{i,k}^2} \right) dy.$$

Formulation of Optimal Control Problem

Control design for RFG

Transition probability matrix

$$\mathbb{P} = \begin{bmatrix} P_{\delta_5^1} \\ P_{\delta_5^2} \\ \vdots \\ P_{\delta_5^5} \end{bmatrix}_{35\times 7}$$

$$P_{\delta_5^2} = \begin{bmatrix} p_{11}(\delta_5^2) & p_{12}(\delta_5^2) & \cdots & p_{17}(\delta_5^2) \\ p_{21}(\delta_5^2) & p_{22}(\delta_5^2) & \cdots & p_{27}(\delta_5^2) \\ \vdots & \vdots & \vdots & \vdots \\ p_{71}(\delta_5^2) & p_{72}(\delta_5^2) & \cdots & p_{77}(\delta_5^2) \end{bmatrix}$$

Per-stage cost function

$$g(x,u) := \lambda_1 \tilde{d}(x, \delta_7^4) + \lambda_2 \tilde{d}(u, \delta_5^3)$$

Reduce the variance of RGF

Less change of VVT degree

$$J_\pi(x_k) = \underset{\substack{w_t \\ t=k,\cdots,k+L}}{E} \sum_{t=k}^{k+L} g(x_t, u_t),$$

Remark: Discrete distance function is defined by $\tilde{d}(\delta_s^i, \delta_s^j) := |i-j|$

The weight coefficients is chosen as $\lambda_1 = 1, \lambda_2 = 0.25.$

Experimental validation for GRF control

Table 1: Engine Specification

Engine type	V6 type 3.5 L
Fuel system	Port & Direct injection
Bore × Stroke (mm)	94 × 83
Compression ratio	11.8:1
Max Power (kW)	228 @ 6400 rpm
Max Torque (Nm)	375 @ 4800 rpm
Displacement (cm^3)	3456

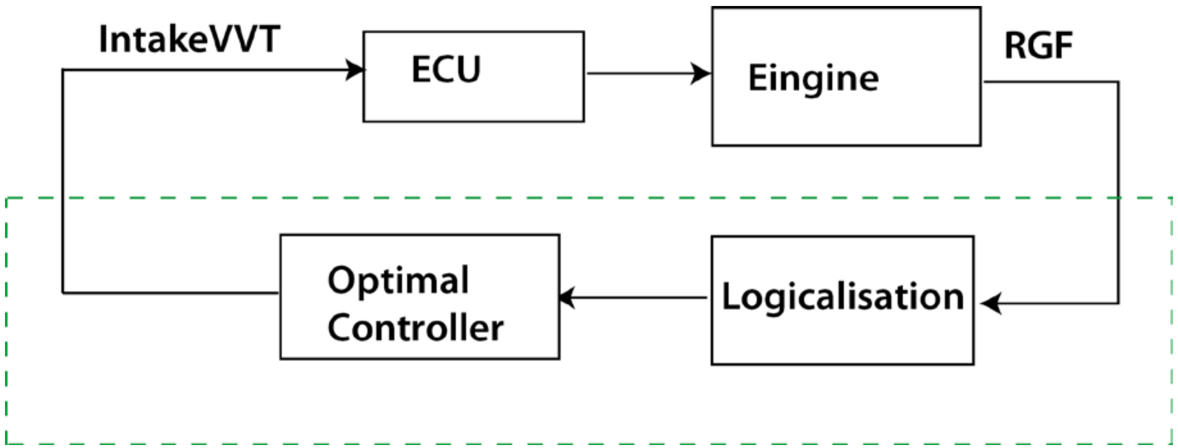


Table 3.2: WORKING CONDITIONS

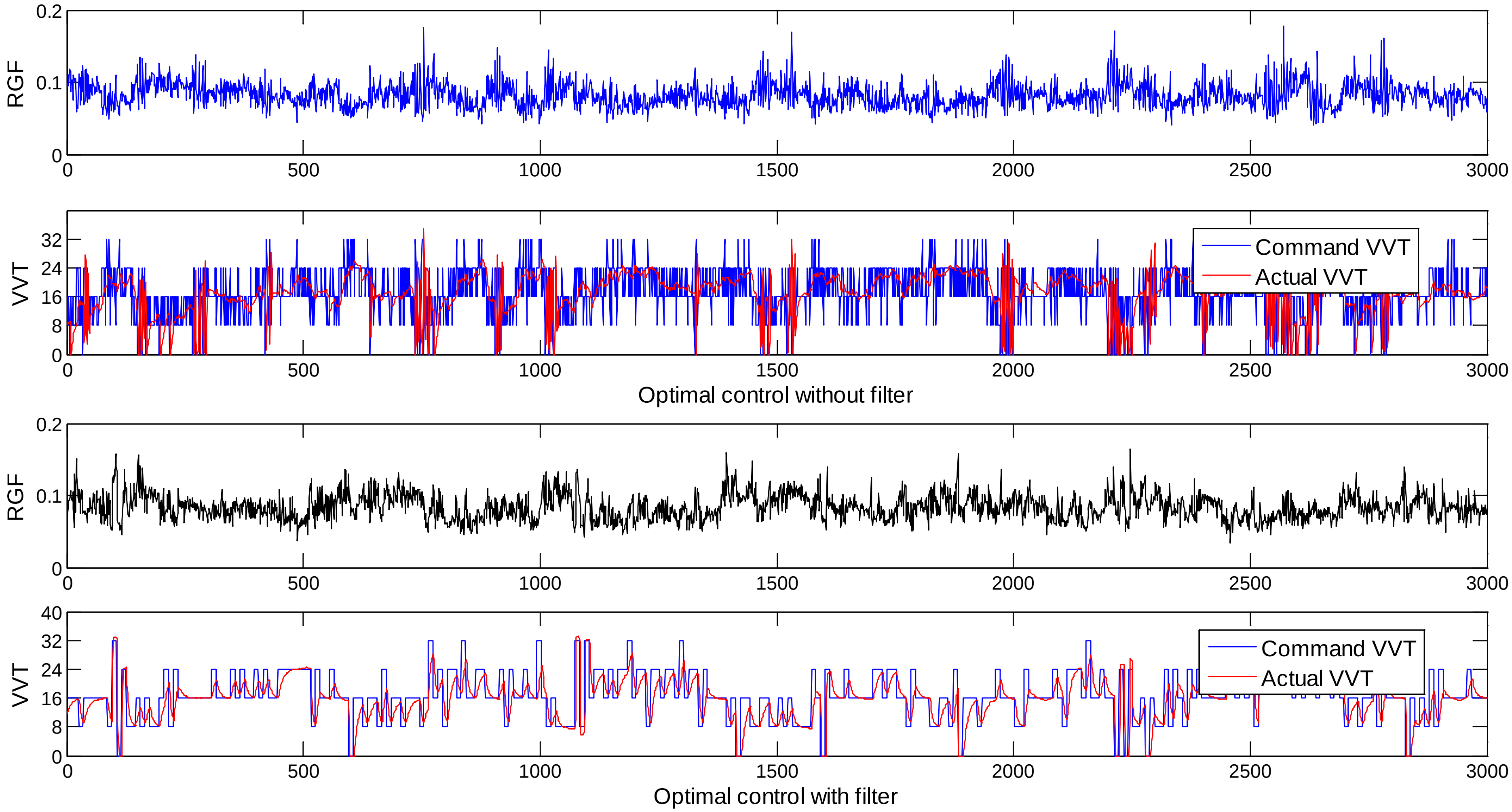
	C_1	C_2	C_3	C_4	C_5	C_6
Engine speed (rpm)	900	900	1200	1200	1500	1500
Load torque (Nm)	90	150	90	150	90	150

The experiment is carried out on the test bench for the 5-th cylinder

Engine speed is 1500 rpm
Water temperature is 353.15 K
Throttle stochastically change from 6.4-10.4 degree

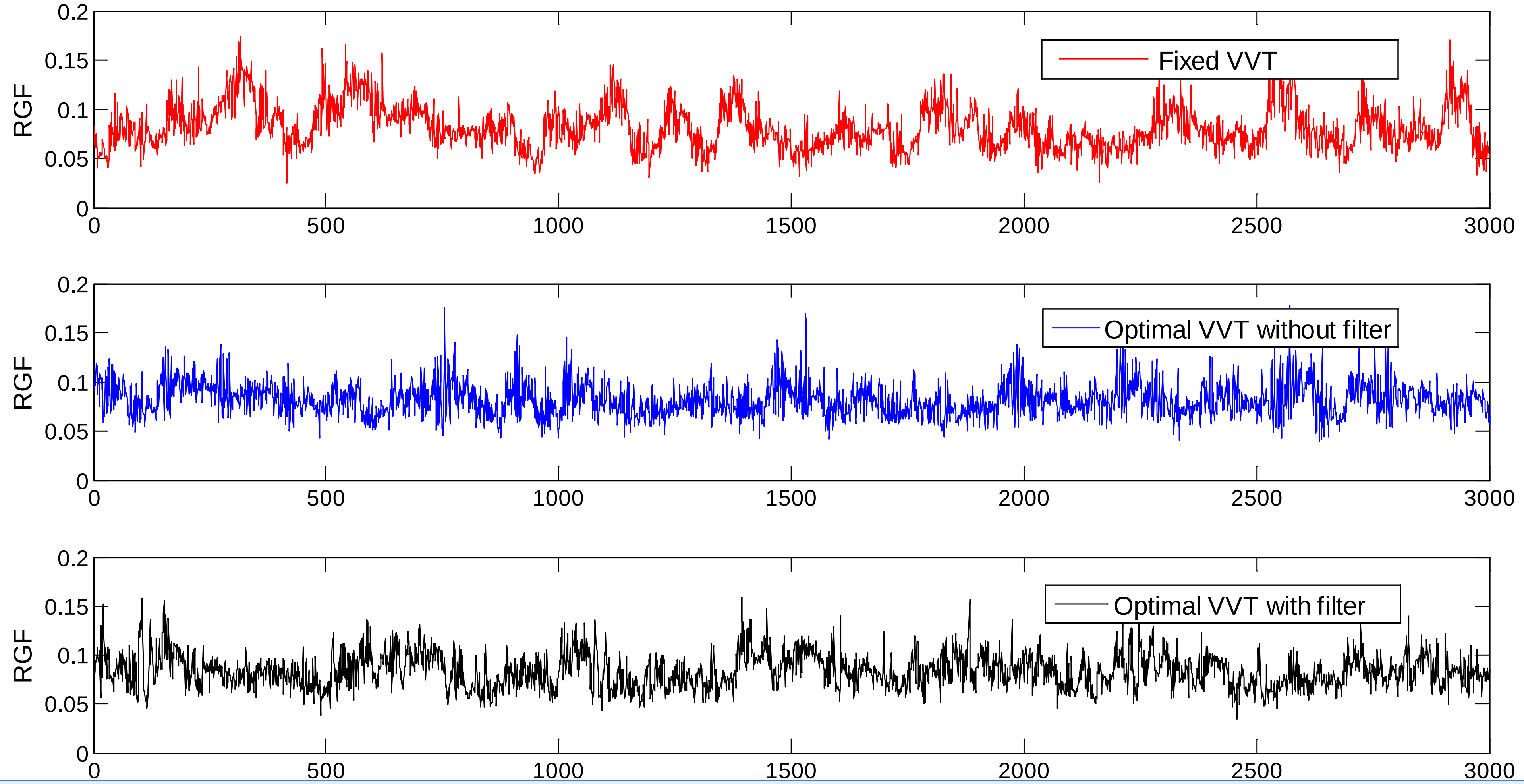
Experimental validation for GRF control

Experimental Result with VVT



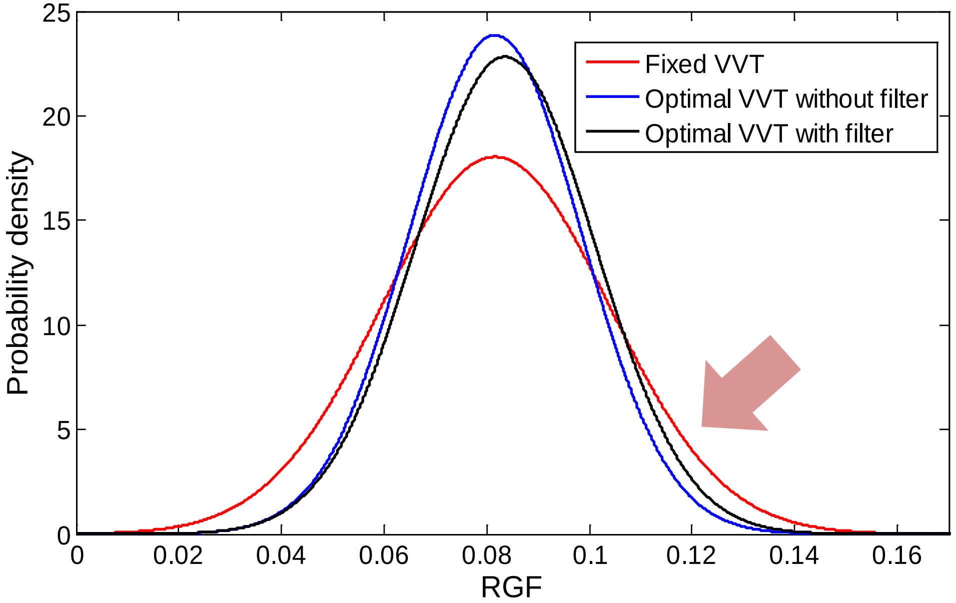
Experimental validation for GRF control

Experimental Result with VVT



[5] **Yuhu Wu**, **Tielong Shen**, Policy iteration approach to control residual gas fraction in IC engines under the framework of stochastic logical dynamics, *IEEE Transactions on Control Systems Technology*, 2016.

Experimental validation for GRF control



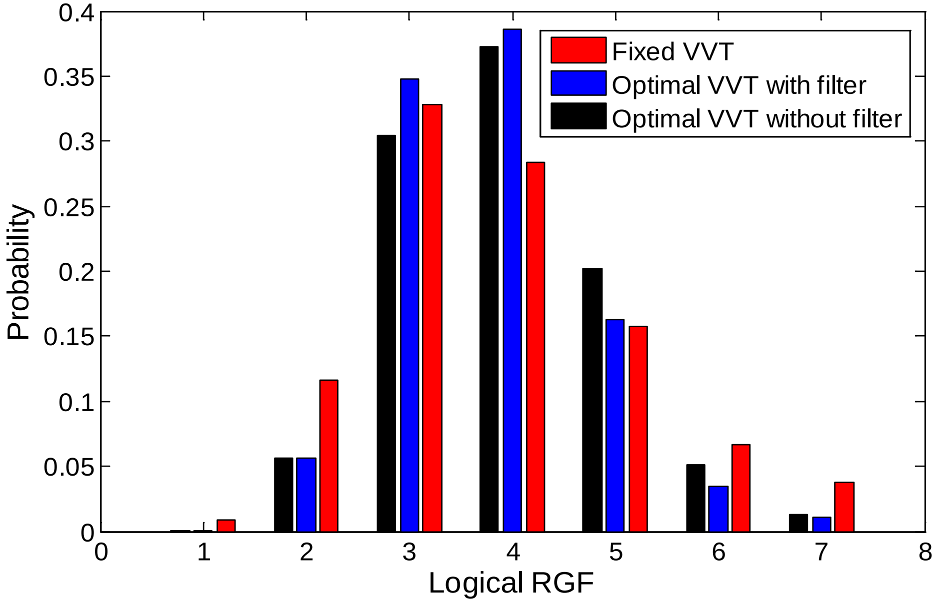


Table 3.2: WORKING CONDITIONS

	C_1	C_2	C_3	C_4	C_5	C_6
Engine speed (rpm)	900	900	1200	1200	1500	1500
Load torque (Nm)	90	150	90	150	90	150

Table 3.3: PERFORMANCE INDEXES

	C_1	C_2	C_3	C_4	C_5	C_6
μ_{open}	0.0917	0.0563	0.0960	0.0626	0.0817	0.0645
σ_{open}	0.0414	0.0271	0.0395	0.0277	0.0221	0.0251
μ_{op}	0.0899	0.0533	0.0833	0.0584	0.0811	0.0631
σ_{op}	0.0386	0.0247	0.0313	0.0246	0.0167	0.0218

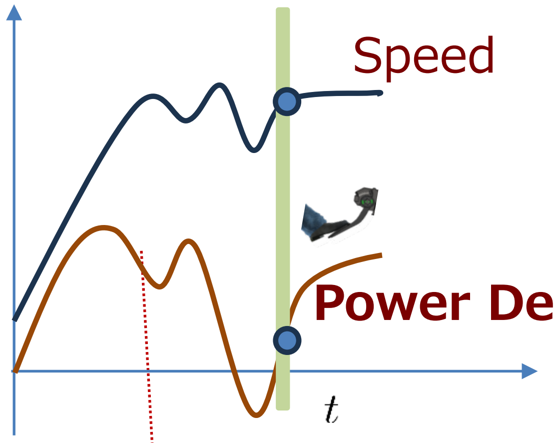
Example of Energy Management of Hybrid Electric Vehicles

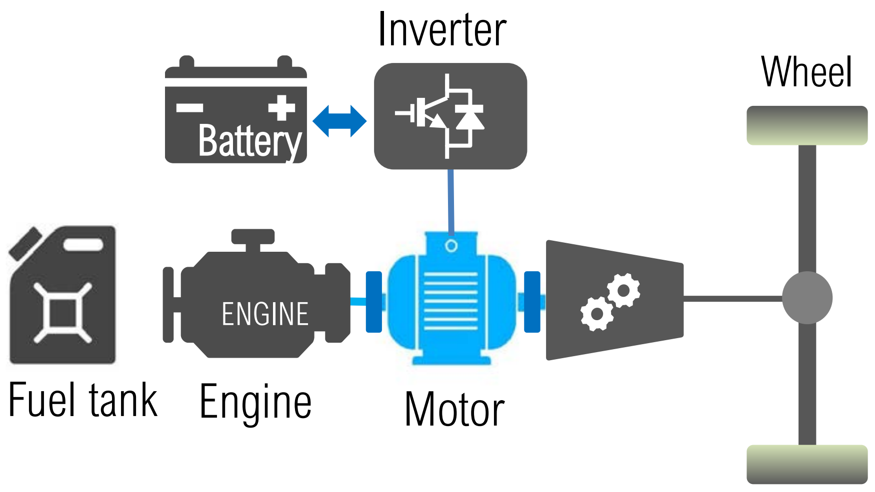
Fundamental Issue in HEV

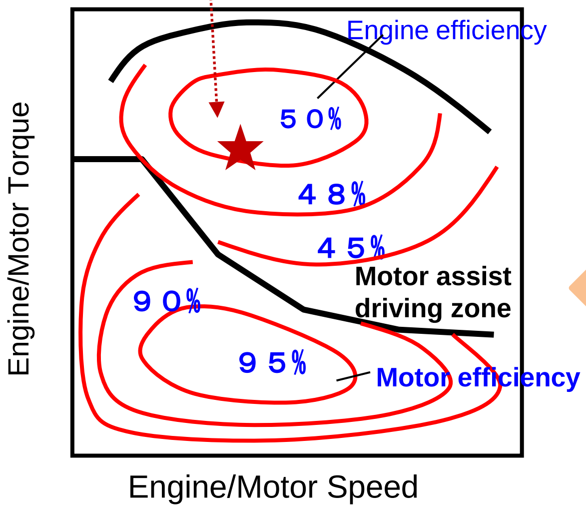
- Constraint by power demand
- Limited SOC有限
- Inertia of powertrain and environment

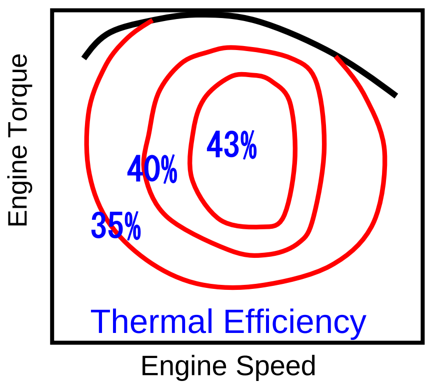
$$\int_0^T \dot{m}_f(\omega,\tau)dt$$

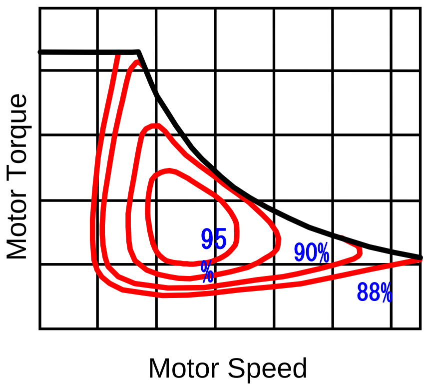
$\dot{m}_f(\omega,\tau)$[mg/s]







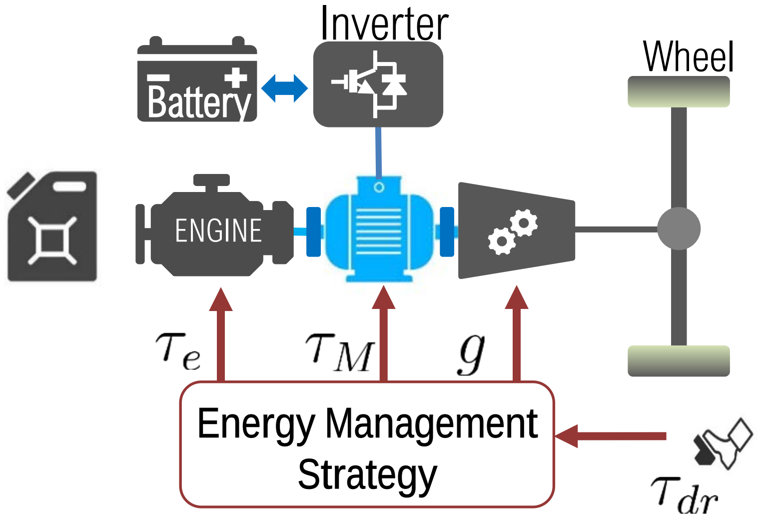




Contents

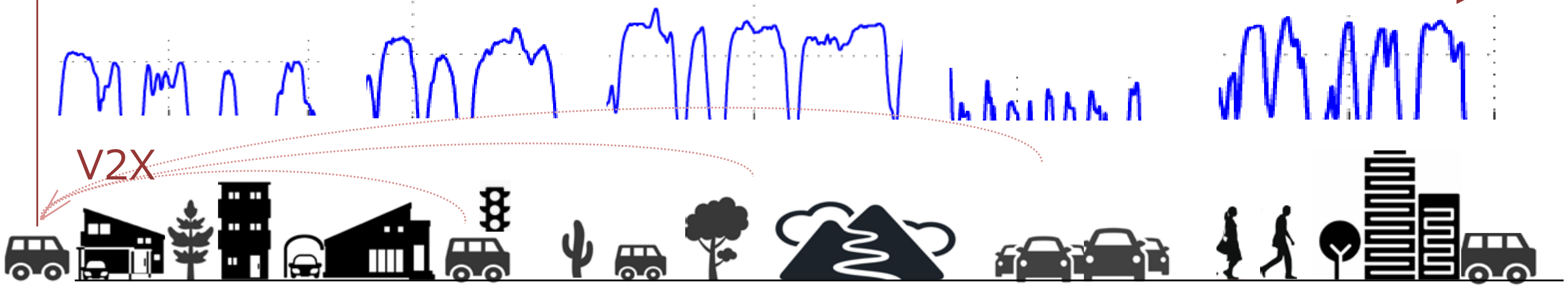
$$\min_{[\tau_e(t),\omega_e(t)]^T} J = \sum_{t=0}^{T-1} \left[r_f m_f(t) + r_e \left(\mathrm{SoC}(t) - \mathrm{SoC}_{ref} \right)^2 \right]$$

$$\text{s.t.} \begin{cases} \mathrm{SoC}(t+1) = \mathrm{SoC}(t) + \Delta t \cdot \Delta\mathrm{SoC}(\tau_m(t), \omega_m(t)), \quad \mathrm{SoC}(0) = \mathrm{SoC}_0 \\ \tau_{emin} \le \tau_e(t) \le \tau_{emax}(\omega_e(t)) \\ \omega_{emin}(v(t)) \le \omega_e(t) \le \omega_{emax}(v(t)) \\ \mathrm{SoC}_{min} \le \mathrm{SoC}(t) \le \mathrm{SoC}_{max} \end{cases}$$



- Uncertainty
- Stochasticity

Long-term prediction



Modeling by Quantization

Finite disjoint interval of state range:

$$S^i = \left[\mathrm{SoC}_{min} + (i-1)\cdot\sigma_x,\quad \mathrm{SoC}_{min} + i\cdot\sigma_x\right),\ i = 1, 2, \cdots, n_x$$

Define $X_n = \{x^1, \cdots, x^n\}$, and $U_m = U_1 \times U_2$, $U_1 = \{u_1^1, \cdots, u_1^{m_1}\}$, $U_2 = \{u_2^1, \cdots, u_2^{m_2}\}$

$$x^i = \mathrm{SoC}_{min} + (i-1)\sigma_x$$

$$u_1^j = \omega_{emin} + (j-1)\sigma_{u_1},\ j = 1, 2\cdots, m_1$$

$$u_2^j = \tau_{emin} + (j-1)\sigma_{u_2},\ j = 1, 2\cdots, m_2$$

$$n_x = \frac{\mathrm{SoC}_{max} - \mathrm{SoC}_{min}}{\sigma_x}$$

$$m_1 = \frac{\omega_{emax} - \omega_{emin}}{\sigma_{u1}}$$

$$m_2 = \frac{\tau_{emax} - \tau_{emin}}{\sigma_{u2}}$$

σ_x: quantized value of state

σ_{u1}, σ_{u2}: quantized value of control variables

Formulation of Optimal Control Problem

Approximate optimal control problem

$$\begin{cases} \mathrm{SoC}(t+1) = \mathrm{SoC}(t) + \Delta\mathrm{SoC}(\tau_m(t), \omega_m(t)) \\ \mathrm{SoC}(0) = \mathrm{SoC}_0 \end{cases}$$

Quantified dynamical system:

$$(\mathrm{QS}) \begin{cases} \hat{x}_n(t+1) = \hat{f}_m^n(\hat{x}_n(t), u_m(t)) \\ \hat{x}_n(0) = q_n(x_0) \end{cases}$$

$$\hat{f}_m^n(x,u) = q_n\left(f(x, q_m(u))\right), \ \forall x \in X_n, u \in U_m$$

$$q_n(x) = \sum_{i=1}^{\Upsilon_n} x_n^i \mathbb{1}_{S_n^i}(x), \quad \forall x \in X$$

$$\mathbb{1}_S(x) = \begin{cases} 1, & x \in S \\ 0, & x \notin S \end{cases}$$

Problem $(AP)_m^n$

Given the quantified dynamical system (QS)
find an optimal control input sequence u_m
such that the corresponding cost functional
reaches the optimal cost

$$J^*(\hat{f}_m^n, x_0) = \inf_{u_m \in U_m} J(\hat{f}_m^n, x_0, u_m)$$

Remark: Solving the problem $(AP)_m^n$ obtains an approximated solution of the original problem.

The demonstrations on the quantitative analysis and convergence of the approximated solution to the accurate one are omitted here.

Problem representation

Proposition

$\forall \;\; n, m \in N$,

$\exists$ a unique logical matrix $\hat{L}^n_m \in L_{n\times m}$

such that quantified dynamics given by (QS) can be expressed in the following linear multi-valued logical dynamics

$$\hat{x}_n(t+1) = \hat{L}^n_m \ltimes \hat{u}_m(t) \ltimes \hat{x}_n(t)$$

Restrict the per-step cost function g on $X_n \times U_m$, then it can be expressed in the form

$$g(x,u) = x^\top G u, \quad \forall x \in \Delta_n, \;\; u \in \Delta_m,$$

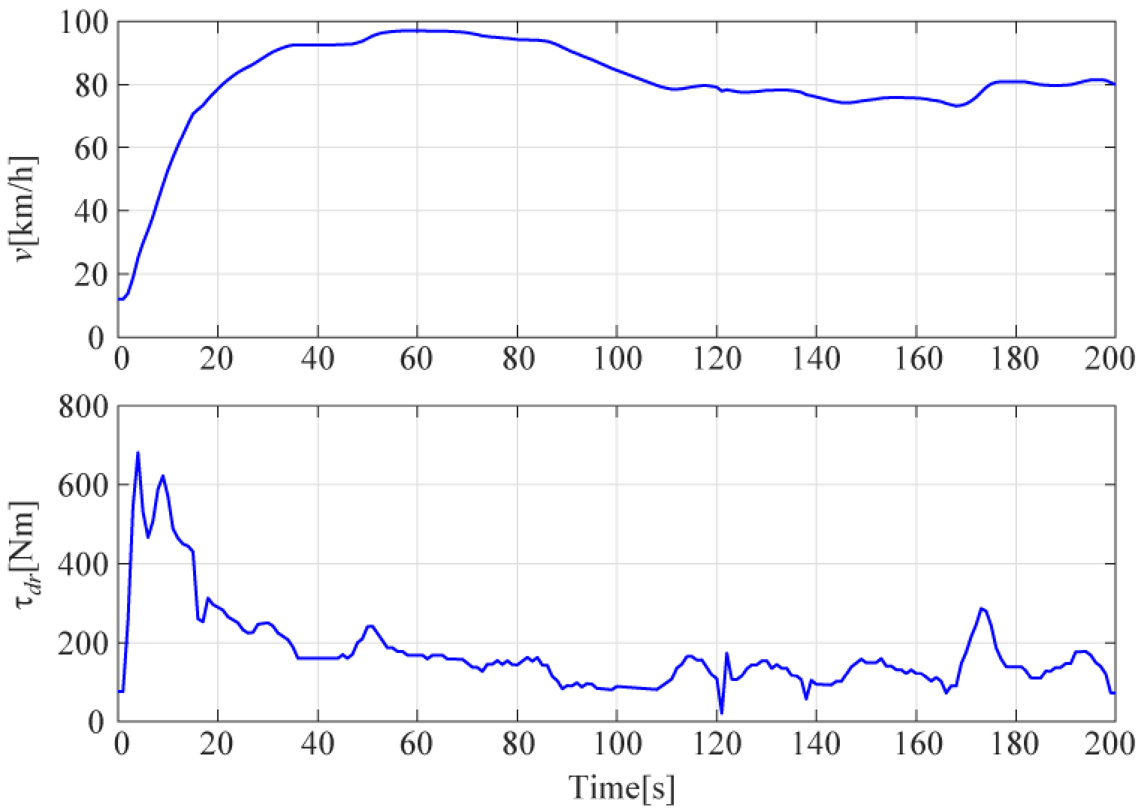
$$G = (G_{i,j})_{\Upsilon_n \times \Upsilon_m} \text{ with } G_{i,j} = g(\delta^i_s, \delta^j_r).$$

Hence, $\forall \; x_0 \in \delta_n$, the objective function becomes

$$J(f, x_0, u) = \sum_{t=0}^{T-1} x(t)^\top G u.$$

Simulation Results

Driving route data



Weighting factors:
$\gamma_f = 1$
$\gamma_e = 800$
SoC_{ref}=0.5

Control period:
$\Delta t = 1[\mathrm{s}]$
Horizon: $T = 200$

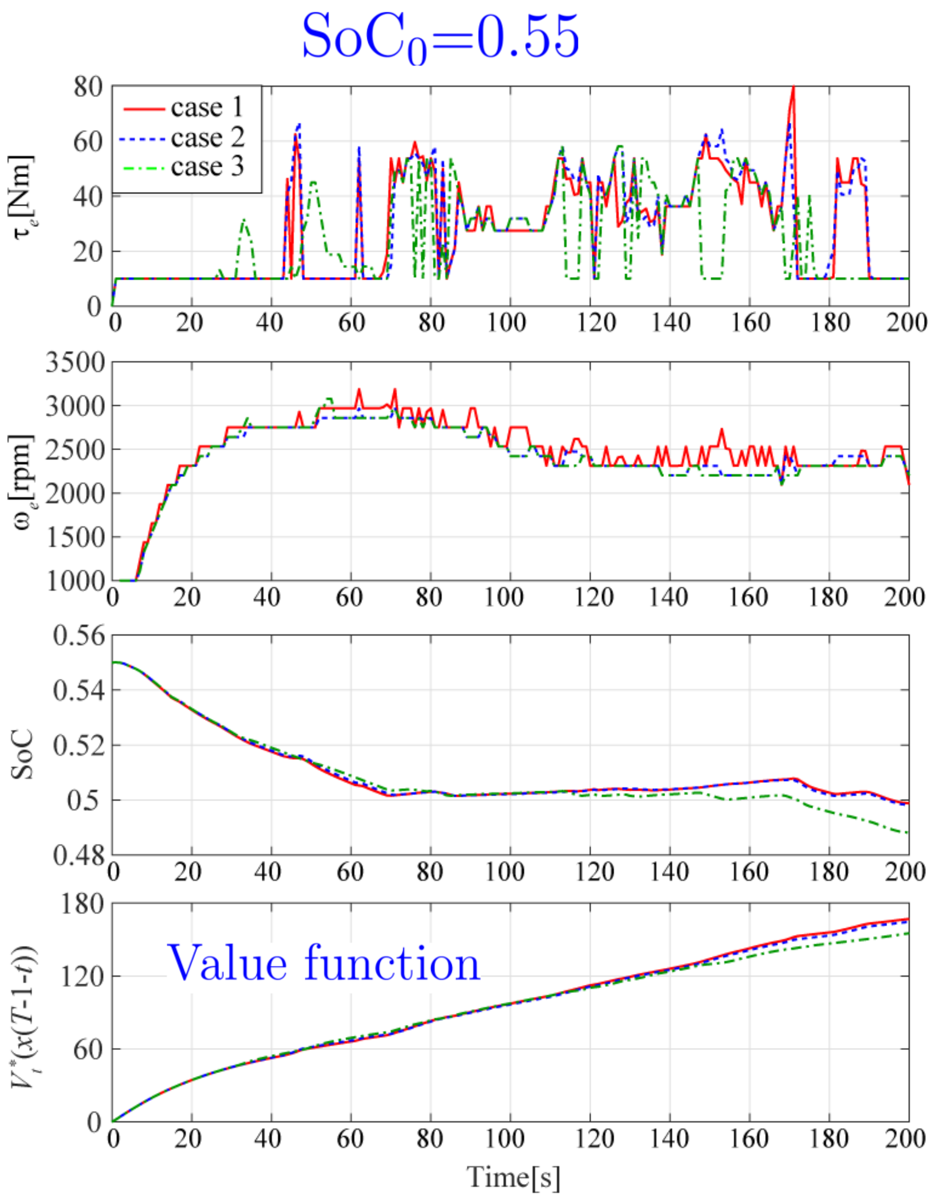
$\mathrm{SoC} \in [0.4, 0.6]$ $\omega_e \in [1000, 4500][\mathrm{rpm}]$

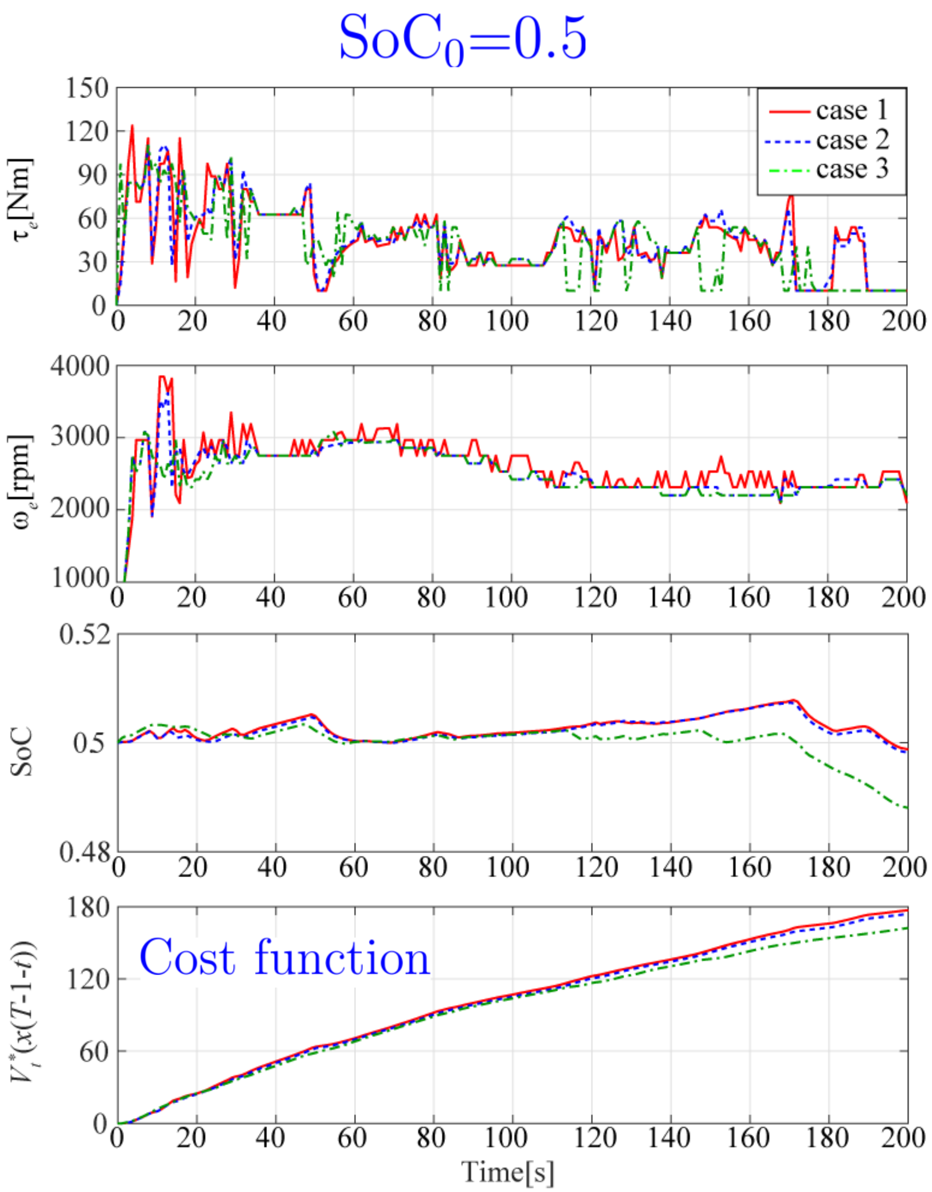
$\tau_e \in [10, 150][\mathrm{Nm}]$

	σ_{SoC}	$\sigma_{\omega e}$[rpm]	$\sigma_{\tau e}$[Nm]
case 1	3.1×10^{-3}	218.75	8.75
case 2	3.1×10^{-3}	109.375	4.375
case 3	7.812×10^{-4}	109.375	4.375

Quantization values of state and control variables

Validation Results





Validation Results

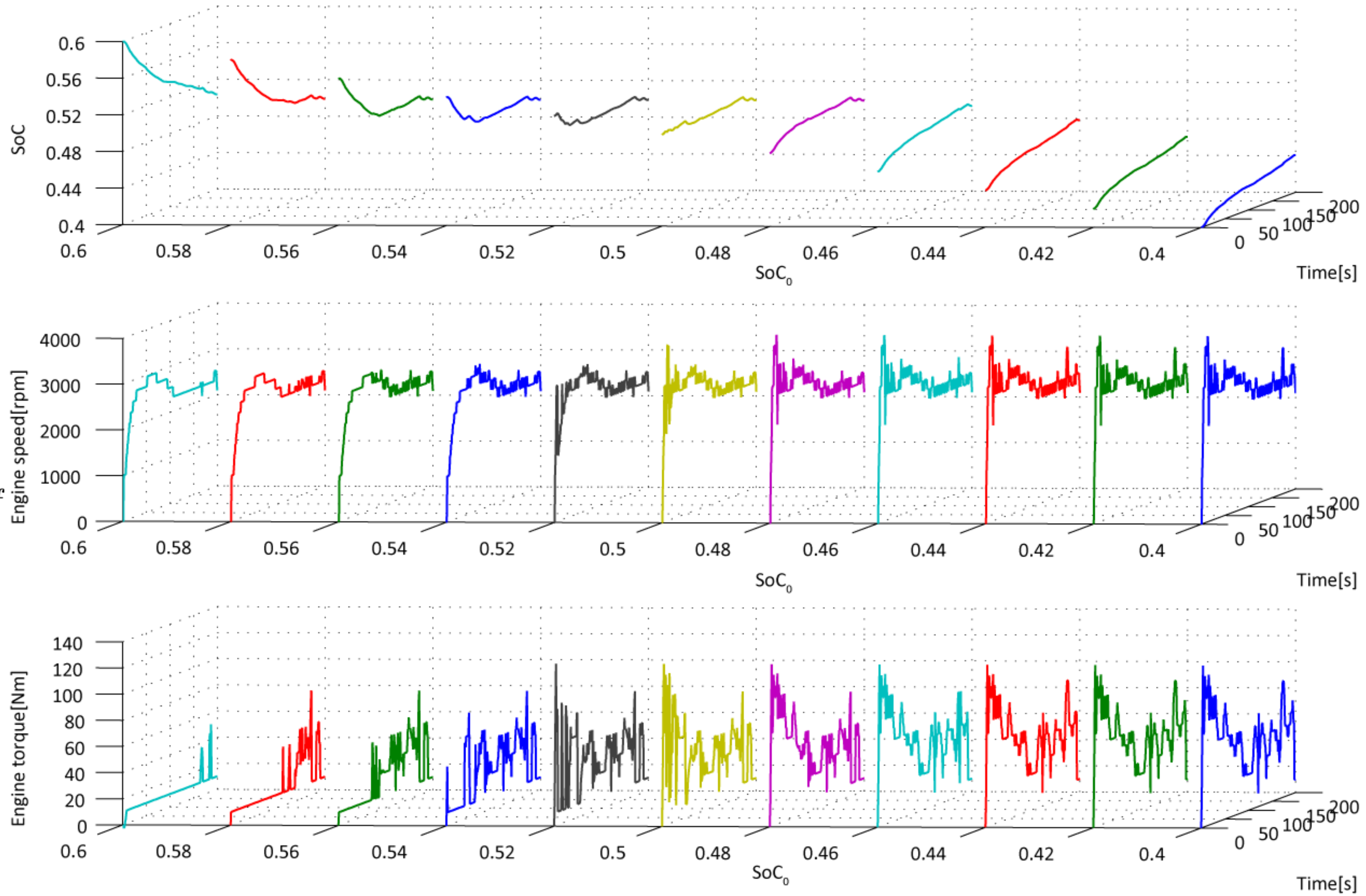
$\mathrm{SoC}_0 \in \{0.4 : 0.02 : 0.6\}$

$\mathrm{SoC} \rightarrow \mathrm{SoC}_{ref},\ \forall \mathrm{SoC}_0$

Each solution guarantees the constraints

If SoC_0 is closed to SoC_{ref}
engine provides more power

otherwise,
motor provides the most demand power



Validation Results

SoC_0		Fuel [L]	Electricity [kWh]	Cost	Computation time [s]
0.55	case 1	0.1636	0.2947	166.952	33.9
	case 2	0.1596	0.2981	164.597	123.1
	case 3	0.1461	0.356	155.1753	504.5
0.5	case 1	0.2388	0.0078	177.1273	33.9
	case 2	0.2299	0.0111	173.9524	123.1
	case 3	0.2144	0.0736	162.3491	504.5

When take smaller quantification factors in terms of both the state and control variables, less cost is achieved, moreover, the fuel economy is improved.

Special Thanks to

TOYOTA

Research on Data Propagation under connected Environment

SIP 革新的燃焼技術
Innovative Combustion Technology
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Cross-ministerial Strategic Innovation Program

On-board optimization algorithm for super lean burn engines

[1] Wu Yuhu, M Kumar, Tielong Shen, A stochastic logical system approach to model and optimal control of cyclic variation of residual gas fraction in combustion engines, Applied Thermal Engineering, 2016.

[2] JunYang, Tielong Shen, and Xiaohong Jiao, “Model-based stochastic optimal air–fuel ratio control with residual gas fraction of spark ignition engines,” IEEE Trans. Control Systems Technology, vol. 22, no. 3, pp.896–910, 2014.

[3]Yuhu Wu, Jiangyan Zhang and Tielong Shen, Logical Network-based Approximated Optimal Control of Continuous Domain System and Application to HEV, Science China Information sciences. 2023

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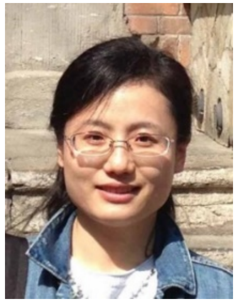

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